

Preface for Instructors

Preface for Students

Acknowledgments

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Fields

Ordered Fields

Completeness

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1B Construction of the Real Numbers: Dedekind Cuts

Exercises 1B

1C Sup and Inf

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1D Open and Closed Subsets of \mathbf{R}

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Exercises 5C

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